FV$^3$-GFDL:
The GFDL Finite-Volume Cubed-sphere Dynamical Core

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\textbf{FV}^3

- Hydrostatic, shallow-atmosphere model (nonhydrostatic version in development)
- Successor to latitude-longitude FV core in NASA GEOS, GFDL AM2.1, and CAM-FV

- GFDL models
  - AM3/CM3
  - HiRAM
  - CM2.5/2.6

- CAM-FV$^3$
  - LASG
  - Academia Sinica
  - GISS ModelE
FV$^3$ Design Philosophy

• Discretization should be guided by physical principles as much as possible
  
  • Finite-volume, integrated form of conservation laws
  
  • Upstream-biased fluxes
  
• Operators “reverse engineered” to achieve desired properties
Development of the FV$^3$ core

- Lin and Rood (1997, QJ): FV shallow-water solver
- Lin (1997, QJ): FV Pressure Gradient Force
- Putman and Lin (2007, JCP): Cubed-sphere advection
- Harris and Lin (in press, MWR): Describes FV$^3$ and grid nesting
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Lin and Rood (1996, MWR)  
 Flux-form advection scheme

\[ q^{n+1} = \frac{1}{\pi^{n+1}} \left\{ \pi^n q^n + F \left[ q^n + \frac{1}{2} g(q^n) \right] + G \left[ q^n + \frac{1}{2} f(q^n) \right] \right\} \]

- 2D scheme derived from 1D PPM operators

- Advective form inner operators f, g, allow elimination of leading-order deformation error
  - Allows preservation of constant tracer field under nondivergent flow

- Flux-form outer operators F, G ensure mass conservation

- All operators must be the same form to avoid Lauritzen instability
Lin and Rood (1996, MWR)
Flux-form advection scheme

• PPM operators are upwind biased
  • More physical, but also more diffusive

• Monotonicity/positivity constraint: important (implicit) source of model diffusion and noise control
  • Nonlinear constraint, “adapts” to flow state

• Scheme maintains linear correlations between tracers when unlimited or when monotonicity constraint applied (not necessarily so for positivity)
1D Advection Test

Lin and Rood 1996, MWR
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Lin and Rood (1997, QJ)
FV shallow-water solver

• Solves layer-averaged vector-invariant equations

\[ \frac{\partial \delta p}{\partial t} + \nabla \cdot (\mathbf{V} \delta p) = 0 \]

\[ \frac{\partial \delta p \Theta}{\partial t} + \nabla \cdot (\mathbf{V} \delta p \Theta) = 0 \]

\[ \frac{\partial \mathbf{V}}{\partial t} = -\Omega \hat{k} \times \mathbf{V} - \nabla (\kappa + \nu \nabla^2 D) - \frac{1}{\rho} \nabla p \bigg|_z \]

• \(\delta p\) is proportional to layer mass

• \(\Theta\): not in SW solver but is in full 3D Solver

• Forward-backward timestepping

• PGF evaluated backward with updated pressure and height
Lin and Rood (1997, QJ)
FV shallow-water solver

- Discretization on D-grid, with C-grid winds used to compute fluxes

- D-grid winds interpolated to get C-grid winds, which are stepped forward a half-step for an approx. to time-centered winds

- Two-grid discretization and time-centered fluxes avoid computational modes

\[
\begin{align*}
\frac{\partial \delta p}{\partial t} + \nabla \cdot (V \delta p) &= 0 \\
\frac{\partial \delta p \Theta}{\partial t} + \nabla \cdot (V \delta p \Theta) &= 0 \\
\frac{\partial V}{\partial t} &= -\Omega \hat{k} \times V - \nabla (\kappa + \nu \nabla^2 D) - \frac{1}{\rho} \nabla p \bigg|_z
\end{align*}
\]
FV shallow-water solver: Vorticity flux

- Nonlinear vorticity flux term in momentum equation
- D-grid allows exact computation of absolute vorticity—no averaging!
- Uses same flux as $\delta p$
  - Consistent flux of mass and vorticity improves preservation of geostrophic balance
- Advantages to this form not apparent in linear analyses
FV shallow-water solver: Kinetic Energy Gradient

- Vector-invariant equations susceptible to Hollingsworth-Kallberg instability if KE gradient not consistent with vorticity flux

- Solution: use C-grid fluxes again to advect wind components, yielding an upstream-biased kinetic energy

\[ \kappa^* = \frac{1}{2} \left\{ \mathcal{K}(u^\theta, \Delta t; u^n) + \mathcal{Y}(v^\phi, \Delta t; v^n) \right\}. \]

- Consistent advection again!
FV shallow-water: Polar vortex test

- Note how well strong PV gradients are maintained

Figure 10. Polar stereographic projection (from the equator to the north pole) of the potential vorticity contours at DAY 24 in the ‘stratospheric vortex erosion’ test case at three different resolutions.
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Lin (1997, QJ)
Finite-Volume Pressure Gradient Force

- Computed from Newton's second law and Green's Theorem

\[
\left( \frac{du}{dt}, \frac{dw}{dt} \right) = \frac{1}{\Delta m} (\Sigma F_x, \Sigma F_z)
\]

\[
\Sigma F = \int_C Pn \, ds
\]

\[
\frac{du}{dt} = g \frac{\Sigma F_x}{\Sigma F_z} = g/\tan \gamma
\]
Lin (1997, QJ)
Finite-Volume Pressure Gradient Force

- Errors lower, with much less noise, compared to a finite-difference pressure gradient evaluation

- Linear line-integral evaluation used in example yields larger errors near model top

- Now using fourth-order scheme to evaluate line integrals

Figure 6. As in Fig. 5, but for the finite-volume method.
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Lin (2004, MWR)
Vertically-Lagrangian Discretization

- Equations of motion are vertically-integrated to yield a series of layers
- Layers like shallow-water except $\theta$ is active
- Layers deform freely while horizontal equations integrated
  - Only cross-layer interaction here is through pressure force
Vertical remapping

- To perform vertical transport, and to avoid layers from becoming infinitesimally thin, we periodically remap to an Eulerian vertical coordinate

- Implicit cubic spline for remapping accuracy
  - Implicit in vertical, so no message passing

- Remapping conserves mass and momentum
  - Option to remap total energy as well, as well as to apply an energy fixer

- Vertical remapping is computationally expensive, but only needs to be done a few times an hour
FV³ and the GFDL models

- Terrain following pressure coordinate: $p_k = a_k + b_k p_s$
  - Other coordinates possible eg. hybrid-isentropic

- Divergence damping: the other model dissipation process
  - Fourth-order damping now standard

- Physics coupling is time-split
  - Vertical diffusion implicit and coupled to land/ocean models
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Putman and Lin (2007, JCP)  
Cubed-sphere advection

- Gnomonic cubed-sphere grid
  - Coordinates are great circles
- Widest cell only $\sqrt{2}$ wider than narrowest
  - More uniform than conformal, elliptic, or spring-dynamics cubed spheres
- Tradeoff: coordinate is non-orthogonal
Putman and Lin (2007, JCP)
Non-orthogonal coordinate

• Gnomonic cubed-sphere is non-orthogonal

• Instead of using numerous metric terms, use covariant and contravariant winds

  • Solution winds are covariant

  • Advection is by contravariant winds

  • KE is product of the two
Cubed-sphere edge handling

- Fluxes need to be the same across edges to preserve mass-conservation

- Gnomonic cubed sphere has ‘kink’ in coordinates at edge

- Currently getting edge values through two-sided linear extrapolation

- More sophisticated edge handling in progress
Cubed-sphere scaling

Scaling of Hydrostatic Cubed Sphere Dynamical Core
(25km with 47 levels) on the CRAY-XE6

Model Years/Wall Clock Day

Number of Cores

MPI Ranks=864: OpenMP Threads=1,2,4,8,16
MPI Ranks=1944: OpenMP Threads=1,2,4,8,16
MPI Ranks=3456: OpenMP Threads=1,2,4,8
MPI Ranks=216,864,1944,3456,5400: OpenMP Threads=1
Grid nesting:
Maritime continent 3:1 nest, c90 coarse grid
Stretched-grid aquaplanet
Precipitation
Stretched-grid aquaplanet
Tropical Cyclones

**c720**
- 2681 cyclones
- 513 at 17.5 m/s
- 263 at 32.5 m/s

**c256r3**
- 2866 cyclones
- 722 at 17.5 m/s
- 206 at 32.5 m/s

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